ALGEBRAIC MODELLING

LINEAR RELATIONSHIPS

GENERAL MATHEMATICS PRELIMINARY

NAME ______________________________

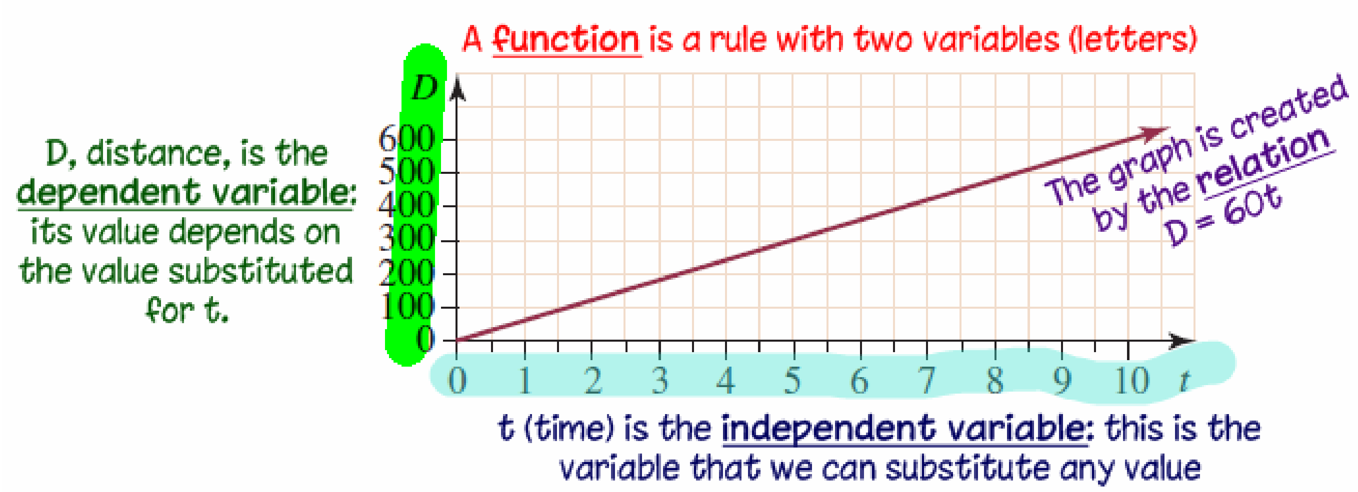
CAPACITY MATRIX – GENERAL MATHEMATICS

TOPIC: ALGEBRAIC MODELLING 2 – Linear Relationships 2 weeks

CONTENT	CAPACITY BREAKDOWN!	DONE IT!!!!	GOT IT!!!!!	ON MY WAY!	WORKING ON IT!	HELP!!!!
1. Review of prerequisite work a. Recognising linear functions b. Gradient of a straight line	Skillsheet 7.1 Skillsheet 7.2					
2. Sketching graphical repS of quantities. 3. Identifying independent and dependent variables in practical contexts	Ex 7A					
4. Gradient and Intercept	Ex 7B Excel Investigating Gradient					
5. Sketching graphs of linear functions expressed in the form y = mx + b	Ex 7C Q1, 2-7a, 8-17 Battleship Functions game Excel Point Gradient					
6. Interpreting linear functions as models of physical phenomena	SkillsheetS 7.3 & 7.4 Ex 7D					
7. Step and Piecewise functions	Ex 7E					
8. Interpreting graphical solution of simultaneous equations drawn from practical situations	Skillsheet 7.5 Excel Sim Eqns Ex 7F					
9. Review (optional)	Chapter review					

Graphing Linear Functions

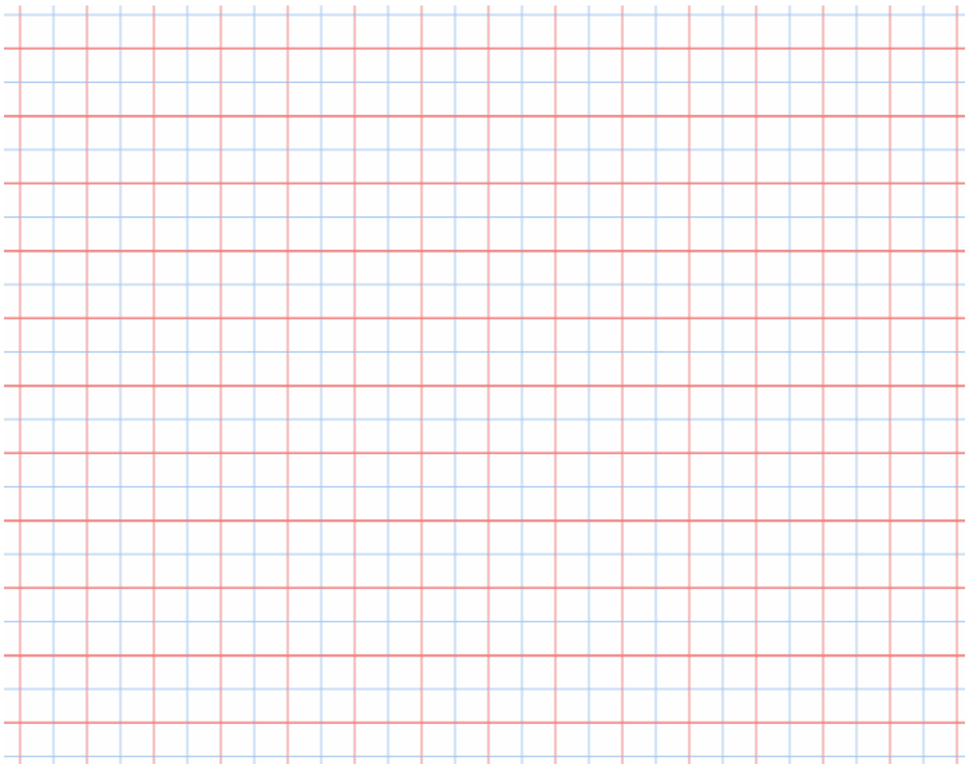
The graph below compares the distance travelled over time by a car travelling at a constant speed of 60 km/h.



A linear function is a graph that, when drawn, is represented by a straight line.

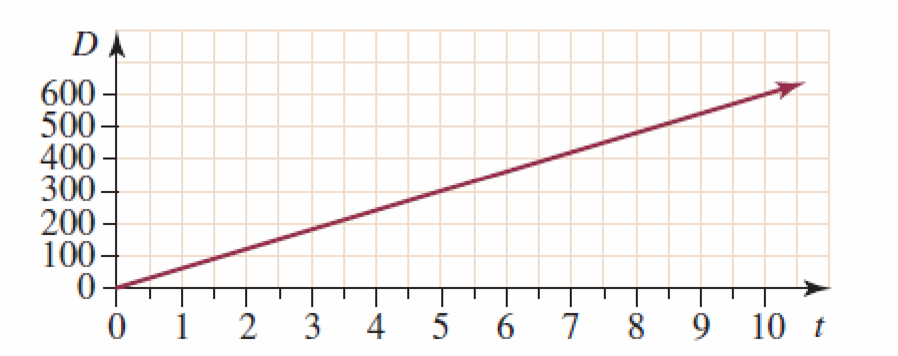
eg The table below shows the amount of money earned by a wage earner:

Hours (H)	10	20	30	40	50
Wage (W)					



GRADIENT AND INTERCEPT

Consider the previous graph:



Select two specific points (if you choose a point where the values are approximate, the process wont work!!)

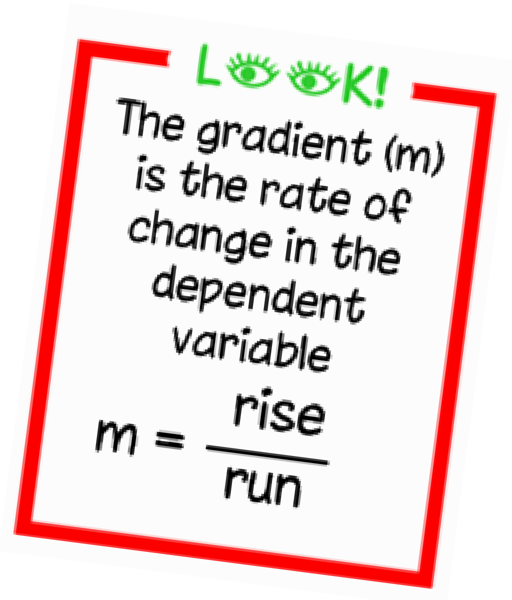
Now create a right triangle using the two points and the function line forming the hypotenuse.

Using the graph scale, calculate:

The vertical height (rise) _______________

The horizontal height (run) _______________

Now calculate the gradient:



$$m = \frac{rise}{run}$$

$m =$ _________

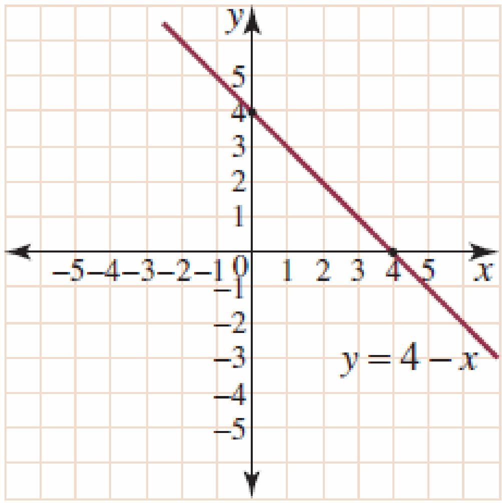
LOOK

If the graph leans to the RIGHT, the gradient is POSITIVE.
It is an INCREASING function.

If the graph leans to the LEFT, the gradient is NEGATIVE.
It is a DECREASING function.

Note where the graph cuts the y-axis _______. This the **Y-INTERCEPT.** This is the initial speed of the car.

eg Calculate the gradient and y-intercept of the function graph below:



eg The table shows the cost of running an excursion for a given number of students:

No. of students	20	40	60	80	100
Cost	\$200	\$300	\$400	\$500	\$600

a) Neatly draw a graph of the cost of this excursion;

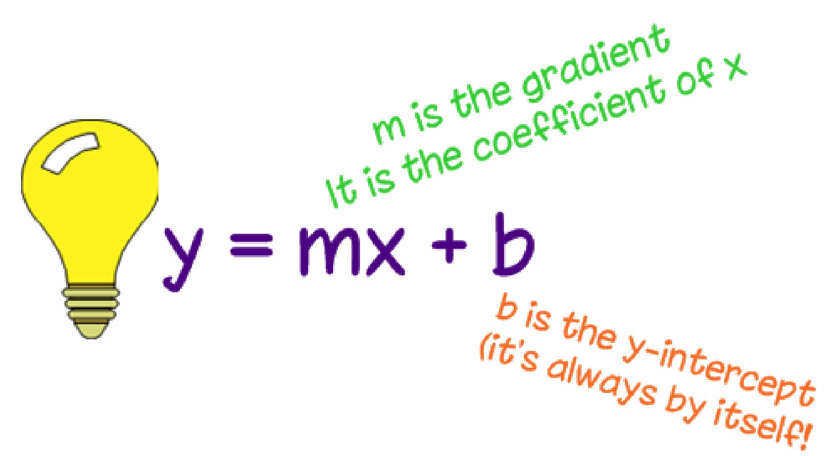
b) Calculate the gradient and explain its meaning in this context;

c) From your graph, find the y-intercept and explain its meaning in this context.

GRADIENT INTERCEPT FORM

INVESTIGATION: Play with the Excel sheet linear_graphs and note the relationship with the formula as you change the gradient and y-intercept values.

Any linear function can be written in the form y = mx + b where m = gradient
b = y-intercept

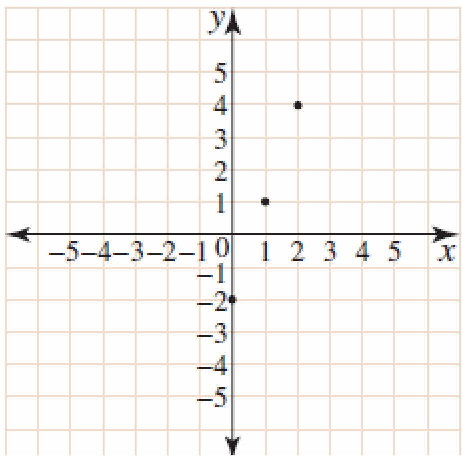


eg Find the gradient and y-intercept of:

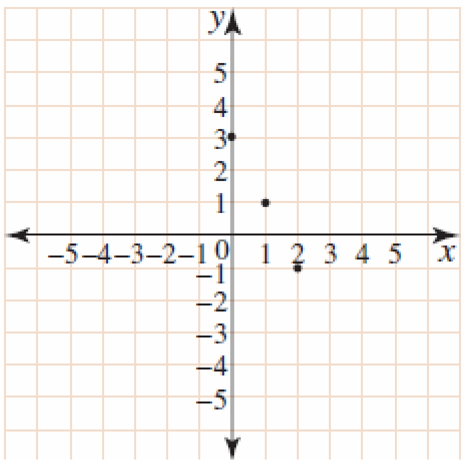
a) y = 3x – 4
b) y = 7 – 4x
   a. 2y = x + 6

eg Sketch the graph of y = 3x – 2

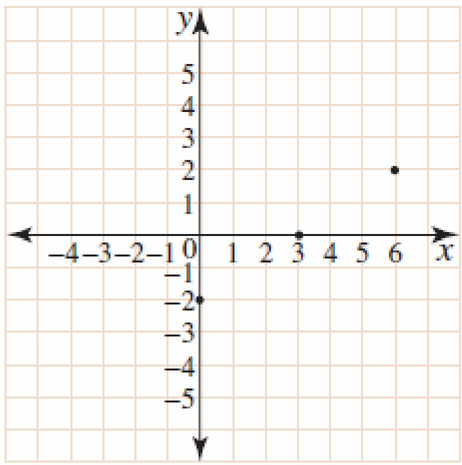
1. List the gradient (in fraction form) and y – intercept;
2. Mark in the y-intercept;
3. FROM THE Y-INTERCEPT, count the rise and then count the run (NOTE: the run always travels to the right!!)
4. Accurately join the two points with a straight line



eg Sketch the graph y = 3 – 2x



eg Sketch $y = \frac{2x}{3} - 2$



SUMMARY:

HOW TO GRAPH?

If you ARE NOT asked for a table of values OR to plot the graph, then…

1. Write down the gradient and the y-intercept;
2. Mark the y-intercept on the y-axis;
3. From the y-intercept, complete the gradient (REMEMBER: you rise (or fall) first, then you ALWAYS RUN TO THE RIGHT.

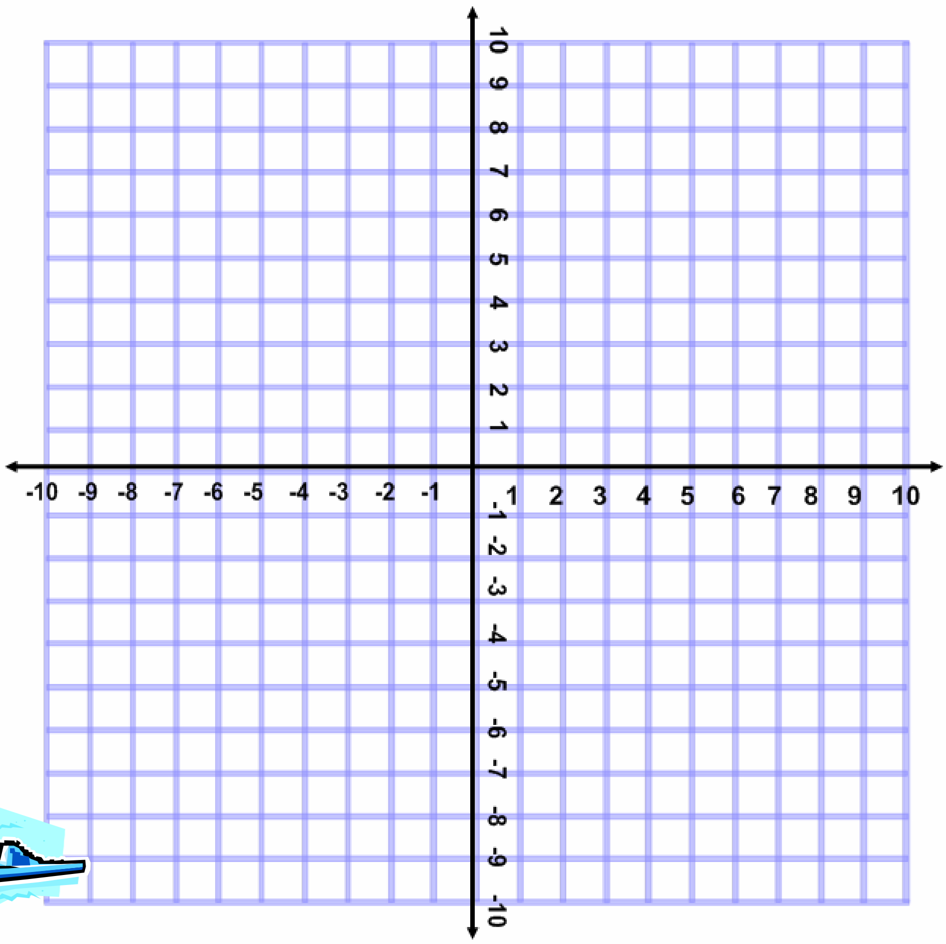
THINGS TO CONSIDER:

1. Precision! If it's rough, you will lose marks (a sketch does not suggest careless work)
2. A negative gradient means the line __________________ – so CHECK!
3. A positive gradient means the line ___________________ - so CHECK!
4. If there is no y- intercept then ____________________________________
5. If the function does not have an x variable (eg $y = 5$) then the line ________________. The gradient of the line is ____________
6. If the function does not have a y variable (eg $x = 5$) then the line _________________. The gradient of the line __________________.
7. The function must always have y as the subject. If not, you must manipulate it so it has, eg Given $2y = 4 - x$, define the gradient and the y-intercept.
8. The function may be given in a form such as $y = \frac{4x-2}{3}$. To calculate the gradient and y-intercept you should __________________________
9. The function that describes the x-axis is __________. The function that describes the y-axis is _________ .

Battleship functions

DIRECTIONS:

- Mark 1 of each ship on the Cartesian plane (use pen):
    - Air craft carriers (5)
    - Cruiser (4)
    - Frigate (3)
    - Patrol boat (2)
    - Raft (1)
- You will be given a function which you will list, noting the gradient and y-intercept. You will then graph the function on the plane.



	FUNCTION	GRADIENT	Y-INTERCEPT
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			

Graphing variations

A variation occurs when one quantity is **proportional** to another.

Eg The number of cars produced on an assembly line varies directly with the number of workers employed on the line. 20 workers can produce 30 cars per week.

The cars are the dependent variable: the number of cars build depends on the number of workers.

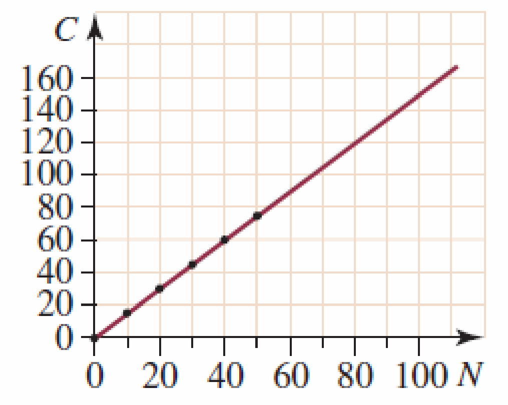
$$m = \frac{\text{the number of cars}}{\text{the number of workers}}$$

a) What is the proportionality of the assembly line? ___________________

The table of values would be:

No. of workers (N)	No. of cars produced (C)
10	15
20	30
30	45
40	60
50	75

The graph would be:



LOOK

In any example where one quantity varies directly with another, the graph will be linear passing through (0, 0).
To draw the function, we only need one other point.
This is known as a DIRECT LINEAR VARIATION.

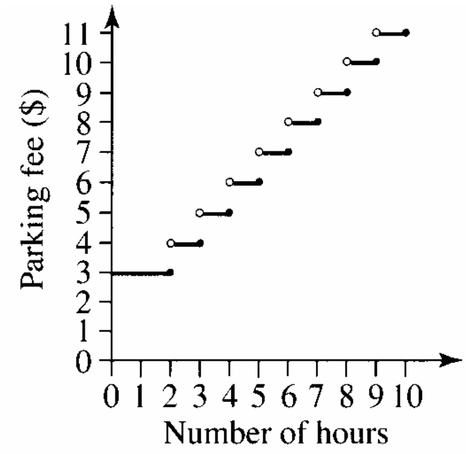
eg The distance travelled by a car is directly proportional to the speed at which it is travelling. If the car travels 285 km in 3 hours, draw a graph of distance travelled against time.

Step and piecework functions

A **step function** is a linear function for which the rule changes as the value of the independent variable changes.

Consider the following graph.

The charge to park is



From the graph, answer the following questions:

What is charged for the following parking times:

1. 3 hours;
2. 2.5 hours;
3. 5 hours;
4. 8 hours, 15 minutes.

INTERSECTION OF LINES

To approximate the simultaneous solution of two linear functions (straight lines) simply graph the two lines and find the point where they meet – the coordinates of that point is the solution.

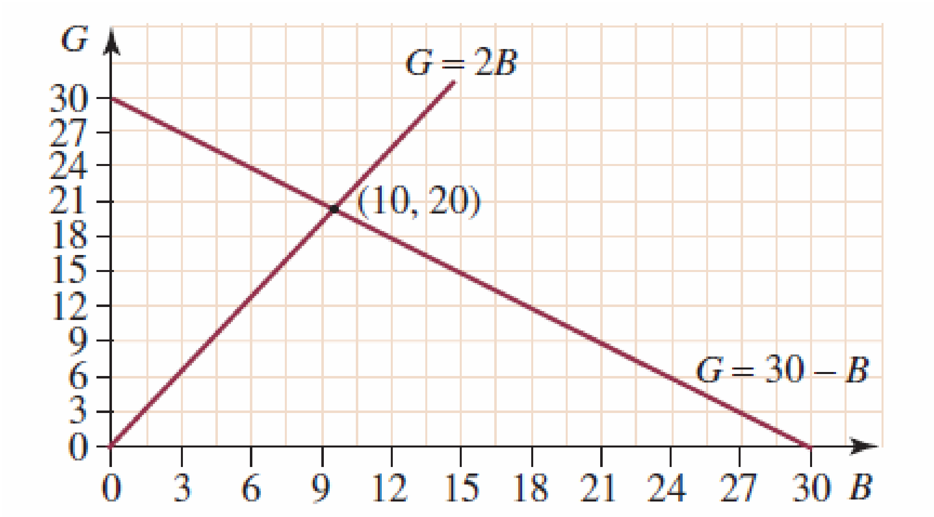
Consider the following:

A class has 30 students. There are twice as many girls as boys. How many boys and girls are in the class?

We can model this by creating 2 linear relationships:

G = 30 – B   and   G = 2B

Then we can graph the two relationships



The solution is the intersection of the two lines.

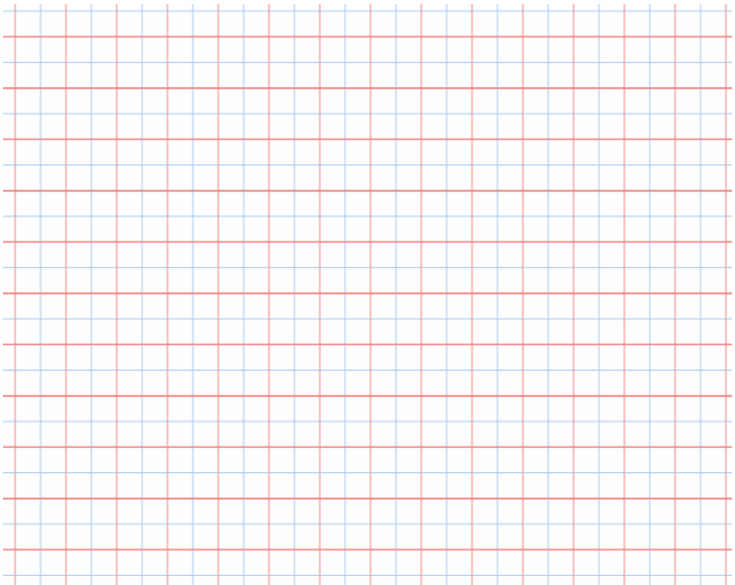
The point of intersection is ________

Therefore the solution is _____ boys and _______ girls

eg Mary is comparing the plans of two telephone companies providing long distance calls. OziExpress One has a monthly access fee of $8.00 and charges $0.70 per call while Optel Easy has a monthly access fee of $10.00 and charges $0.50 per call.

a) Represent the costs C of both plans as linear functions;

b) Graph both functions on the same axes for values of n from 0 to 20.



c) If Mary usually makes 8 long distance calls per month, which plan is the better one for her?

d) For what number of calls per month do both plans charge the same cost?

e) To what type of caller would you recommend:

   i. The OziExpress plan?

   ii. The Optel Easy plan.